Final Exam Review

These review slides and earlier ones found linked to on BlackBoard

Bring a photo ID card:
    Rocket Card, Driver's License

Exam Time

TR class

Monday December 9

12:30 – 2:30

Held in the regular classroom.

Extra office hours in UHall 3014

Monday 10:00-12:30

Covers:

12.1 Counting Methods
12.2 Fundamental Counting Principle
12.3 Permutations and Combinations
13.1 The Basics of Probability
13.2 Complements and Unions of Events
13.3 Conditional Probability
14.1 Organization and Visualizing of Data
14.2 Measures of Central Tendancy
14.3 Measures of Dispersion
14.4 The Normal Distribution

Know the basic vocabulary of the sections.

The test will be multiple choice.

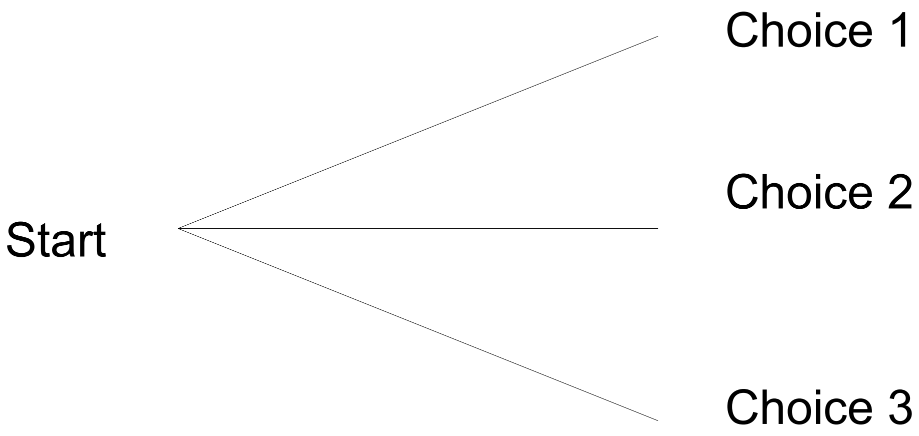
The test will be like the online HW rather than the lab assignments.

In a selection where repetition is allowed, the phrase **with repetition** is used.

In a selection where repetition is not allowed, the phrase **without repetition** is used.

There is a graphical way to organize and count.

A **tree diagram** is a visual method for each new choice at a step we get a new branch. Work from left to right.



Choice 1

Choice 2

Start

Choice 3

Example: A bag has R, G, B, Y marbles. Draw the tree diagram for removing 2 without replacement.

Example: A bag has R, G, B, Y marbles. Draw the tree diagram for removing 2 with replacement.

THE FUNDAMENTAL COUNTING PRINCIPLE (FCP) If we want to perform a series of tasks and the first task can be done in a ways, the second can be done in b ways, the third can be done in c ways, and so on, then all the tasks can be done in $a \times b \times c \times \cdots$ ways.

At an Ice Cream shop they have 5 different flavors of ice cream and you can pick one of 4 toppings.

How many choices do you have?

At an Ice Cream shop they have 5 different flavors of ice cream and you can pick one of 4 toppings.

How many choices do you have?

5 choices of flavors,
4 choices of toppings

5x4 = 20

Example: A license plate has 3 letters followed by three numbers. Every letter and number must be unique.

How many different license plates are there?

Example: A license plate has 3 letters followed by three numbers. Every letter and number must be unique.

How many different license plates are there?

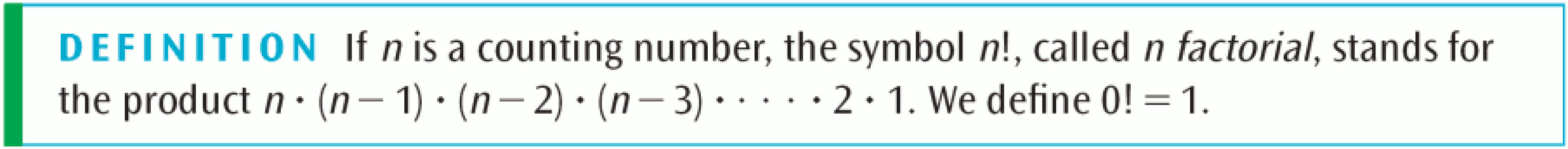
$\underline{26} \times \underline{25} \times \underline{24} \times \underline{10} \times \underline{9} \times \underline{8}$

$= 11{,}232{,}000$

A permutation of n objects r at a time is the number of ways r things (out of n) can be chosen in an ordered way.

A combination of n objects r at a time is the number of ways r things (out of n) can be chosen in an unordered way.

Shortcut/Defintion



DEFINITION If n is a counting number, the symbol $n!$, called *n factorial*, stands for the product $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \cdot \cdot \cdot \cdot 2 \cdot 1$. We define $0! = 1$.

Example: 5! = 5x4x3x2x1

FORMULA FOR COMPUTING $P(n, r)$

$$P(n, r) = \frac{n!}{(n-r)!}$$

FORMULA FOR COMPUTING $C(n, r)$ If we choose r objects from a set of n objects, we say that we are forming a **combination** of n objects taken r at a time. The notation $C(n, r)$ denotes the number of such combinations.† Also,

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r! \cdot (n-r)!}.$$

Example: How many ways are there to make a 2 topping pizza if you have 5 toppings to choose from?

Example: How many ways are there to make a 2 topping pizza if you have 5 toppings to choose from?

Order does not matter = combination.

$$C(5,2) = \frac{5!}{(5-2)!\ 2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = 10$$

An **experiment** is a controlled operation that yields a set of results.

The possible results of an experiment are called its **outcomes**. The set of outcomes are the **sample space**.

An **event** is a subcollection of the outcomes of an experiment.

Probability is a fraction (or decimal) between 0 (doesn't happen) and 1 (always happens).

Probability of Event E = P(E)

Theoretical = found mathmatically

$$\frac{\text{number of times event happens}}{\text{number of possible outcomes}}$$

Empirical = found by running experiments

$$\frac{\text{number of times event happens}}{\text{number of times experiment run}}$$

Example: Experiment is roll a die.

Sample space: { 1, 2, 3, 4, 5, 6 }

What is the probability of rolling an odd number?

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Sample space: { 1, 2, 3, 4, 5, 6 }

What is the probability of rolling an odd number?

Event E = { 1, 3, 5 }

P(E) = 3/6 = 1/2

Odds and probability are similar.

Probability
 #(it happens) / #(total)

Odds for
 #(it happens) : #(it does not happen)

Odds against
 #(it does not happen) : #(it happens)

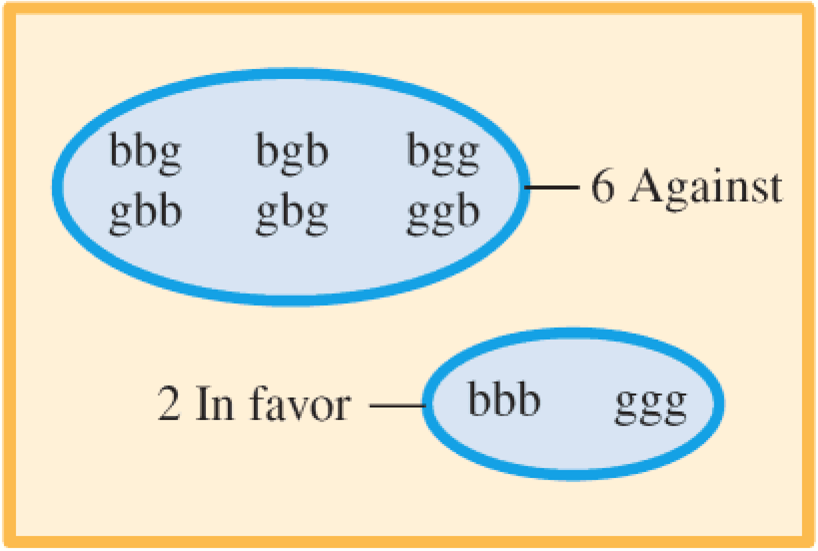
If a family has three children, what are the odds against all three children being the same gender?

If a family has three children, what are the odds against all three children being the same gender?

E = same gender
= { bbb, ggg }

E' = complement = not all the same
= { bbg, bgb, bgg, gbb, gbg, ggb }

6:2 against or
3:1 against



Recall:
The complement of a set is the collection of elements not in that set.

A' = { elements not in A }

The complement of an event E, is the collection of outcomes not in E.

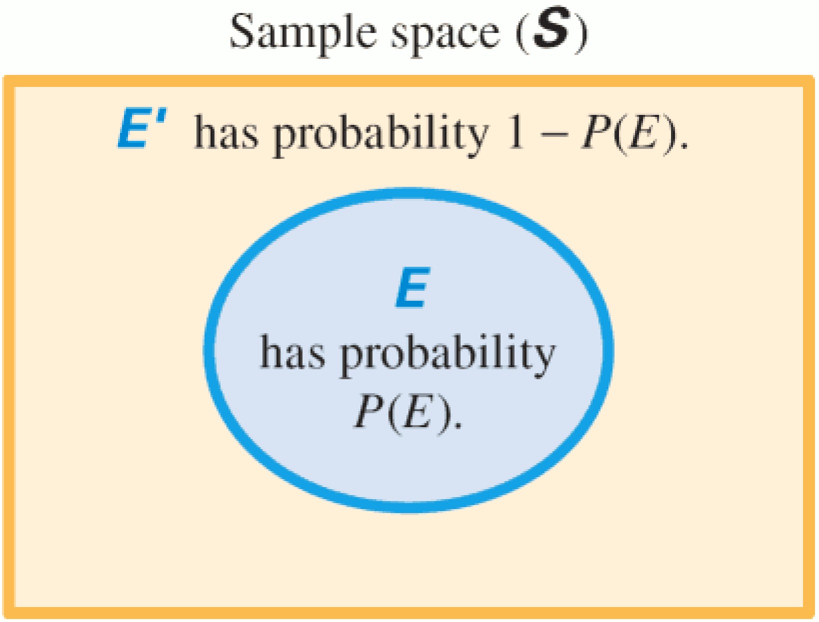
E' = { outcomes not in E }

If an outcome is in the sample space, it must be in E or E'.

So E and E' give all outcomes.

So $P(E) + P(E') = 1$ (100%)

COMPUTING THE PROBABILITY OF THE COMPLEMENT OF AN EVENT If E is an event, then $P(E') = 1 - P(E)$.



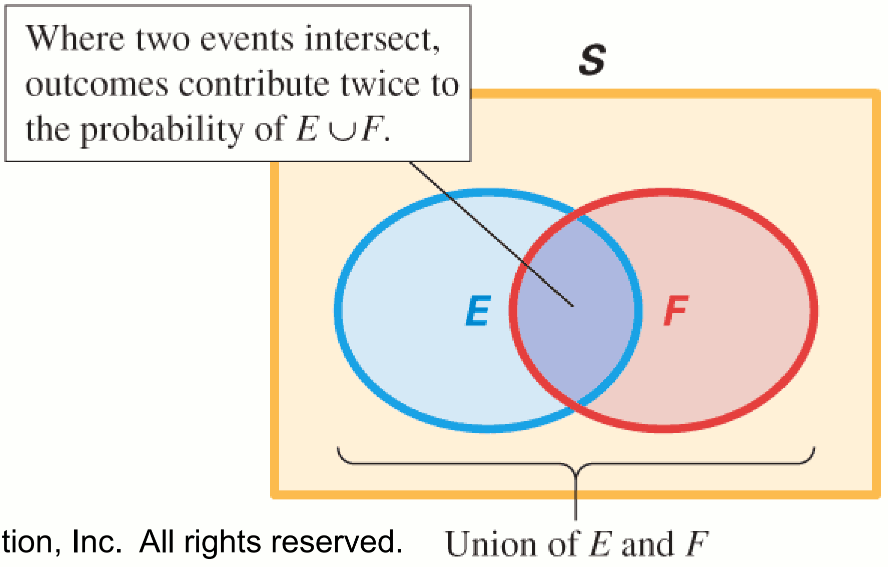
Unions of Events

RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If E and F have no outcomes in common, they are called *mutually exclusive events.* In this case, because $E \cap F = \varnothing$, the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$



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Unions of Events

- Example: If we select a single card from a standard 52-card deck, what is the probability that we draw either a heart or a face card?

- Solution: Let *H* be the event “draw a heart” and *F* be the event “draw a face card.” We are looking for $P(H \cup F)$.

(continued on next slide)

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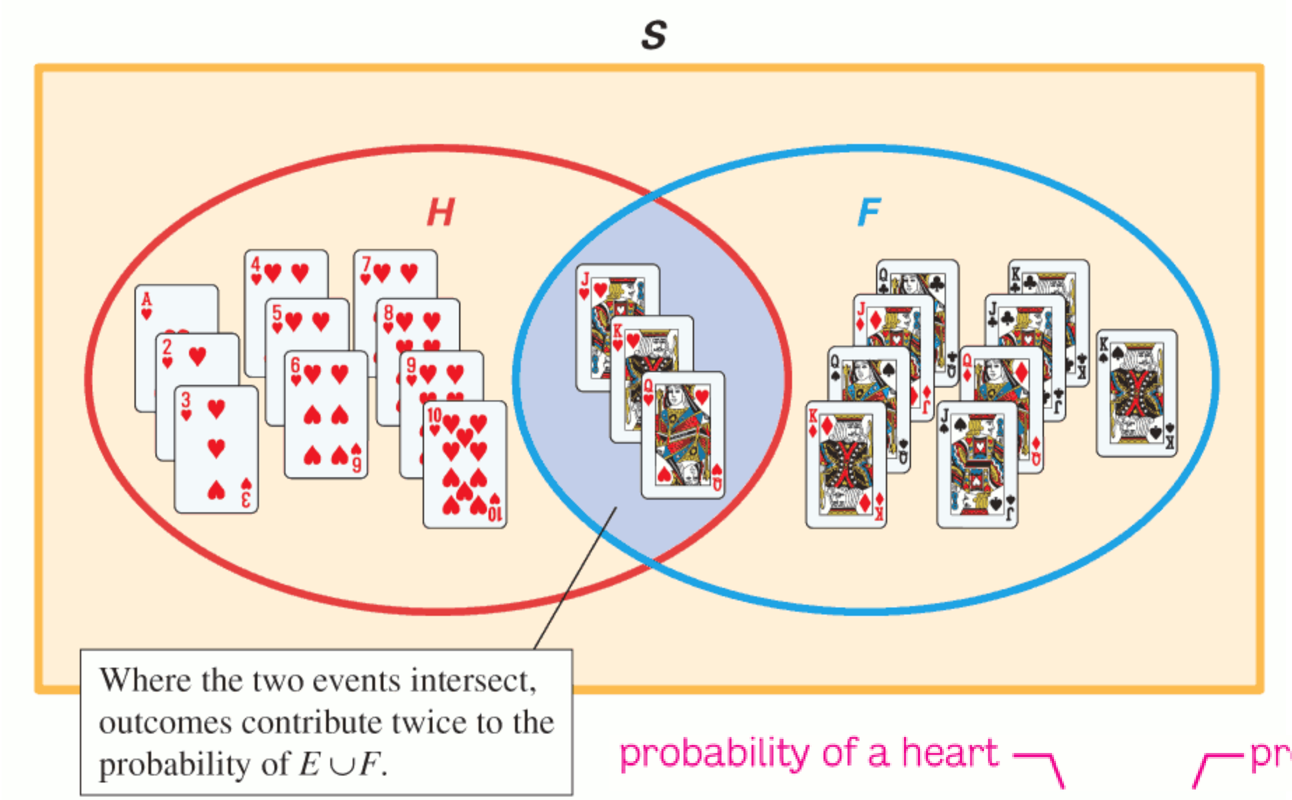
Unions of Events



Where the two events intersect, outcomes contribute twice to the probability of $E \cup F$.

There are 13 hearts, 12 face cards, and 3 cards that are both hearts and face cards.

$$P(H \cup F) = P(H) + P(F) - P(H \cap F) = \overset{\text{probability of a heart}}{\frac{13}{52}} + \overset{\text{probability of a face card}}{\frac{12}{52}} - \underset{\text{probability of a heart that is a face card}}{\frac{3}{52}} = \frac{22}{52} = \frac{11}{26}$$

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If you are given 3 out of the 4 terms in the equation

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Then you can use algebra to find the remaining term.

This can also be read as

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

Conditional probability is the probability of one event (F) happening assuming that another event (E) does.

Examples:

- probability that someone is happy given that they just won $$$.
- probability that someone passes an exam given that they did not study.

The probability that F happens given that E does is denoted P(F|E)

It is read “probability of F given E”

Example: Roll a die for an experiment.

What is the probability it is odd given that the value was a prime number?

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The event assumed to happen was that the value was prime.

Among those the event is when is it odd.

Example: Roll a die for an experiment.

What is the probability it is odd given that the value was a prime number?

The event assumed to happen was that the value was prime.

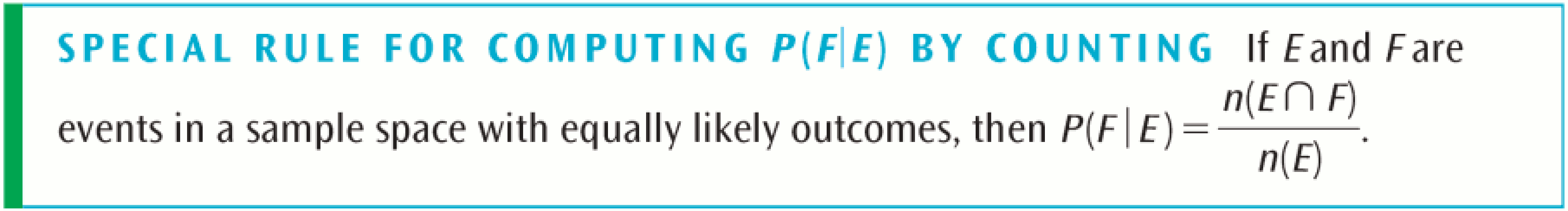
{ 2, 3, 5 }

Among those the event is when is it odd.

{ 3, 5 }

P(odd | prime) = 2/3

The previous examples lead to a way to count P(F | E) by a formula:



SPECIAL RULE FOR COMPUTING $P(F|E)$ BY COUNTING If E and F are events in a sample space with equally likely outcomes, then $P(F \mid E) = \dfrac{n(E \cap F)}{n(E)}$.

Counting how often a data value occurs is its **frequency**.

Counting the percent (%) is its **relative frequency**.

Frequency distribution (or **table**) is the collection of data with its frequency.

Similar for **Relative frequency distribution**.

Data can also be grouped in ranges.

Data: 51 56 58 53 60 53 61 53 59 57
53 56 61 54 58 59 52 55 56 56

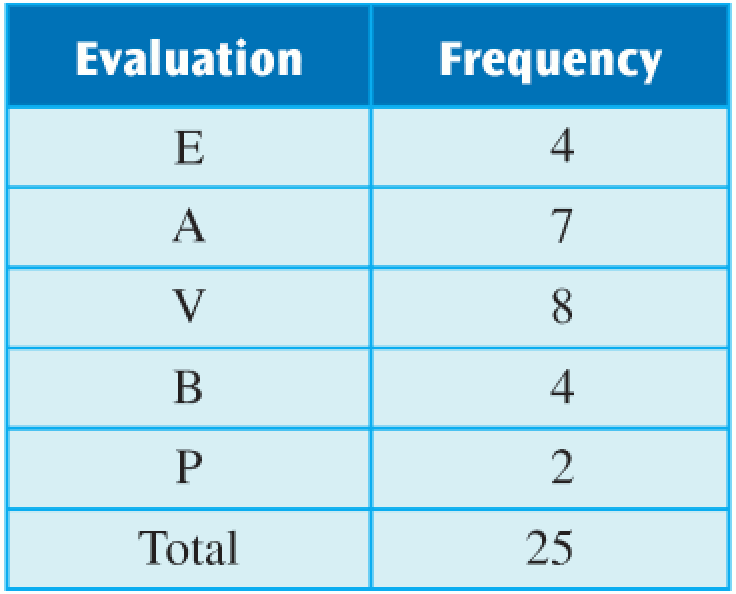
Range	Frequency	Rel Freq
50 – 52	2	10%
53 – 55	6	30%
56 – 58	7	35%
59 – 61	5	25%

Total Data: 20

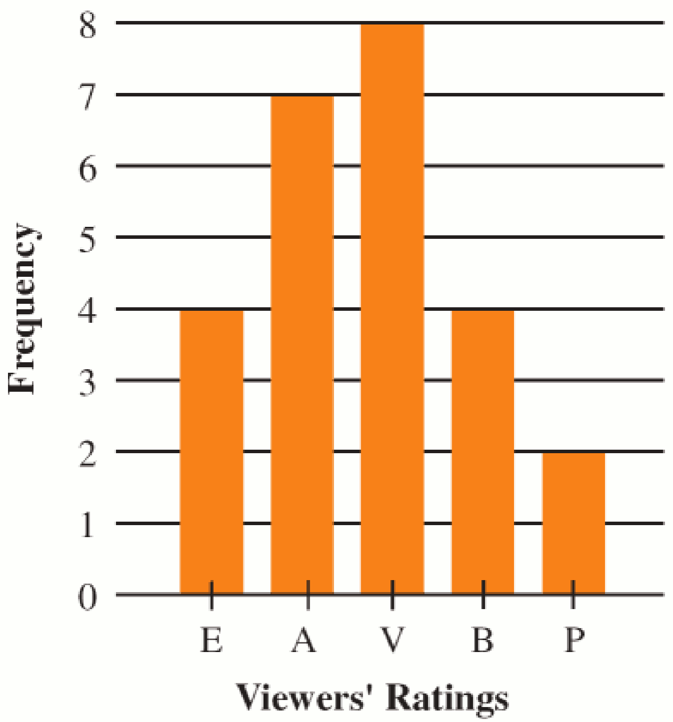
Data Visualizing

A **bar graph** is 2 dimensional

- x-axis is data (or ranges)
- y-axis is (relative) frequency
- draw the height of a rectangle equal to its (relative frequency)



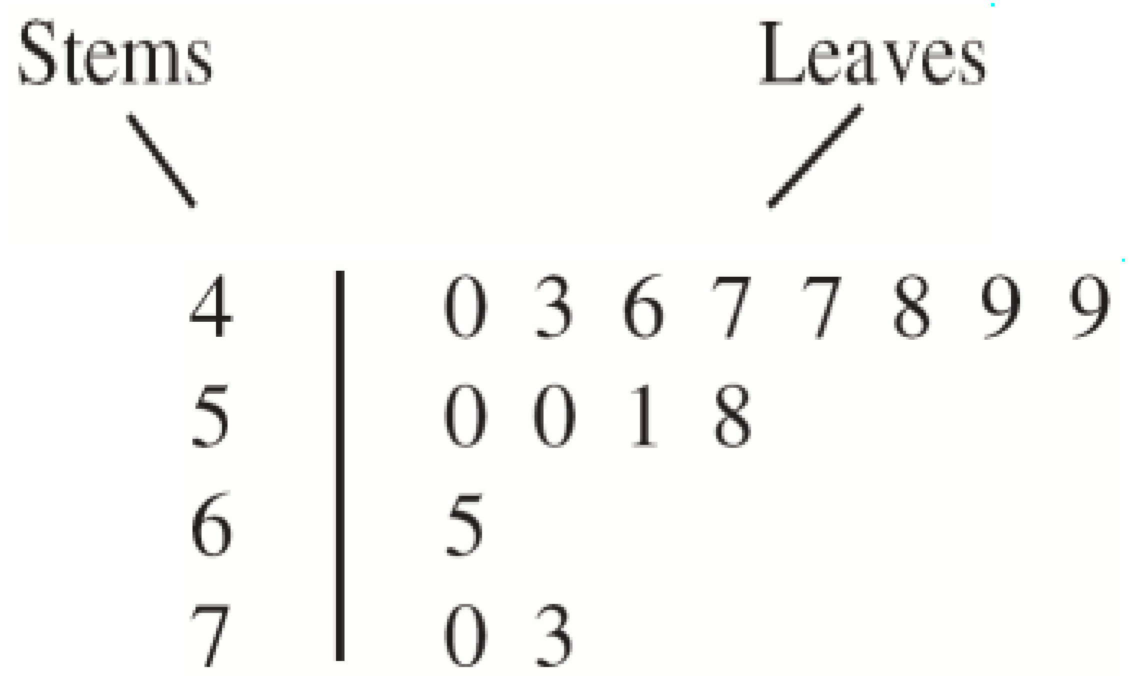
Evaluation	Frequency
E	4
A	7
V	8
B	4
P	2
Total	25



A **stem-and-leaf** plot splits numerical data into two pieces:

- stem = first digit
- leaf = last digit

46 43 40 47 49 70 65 50 73 49 47 48 51 58 50



4 | 0 3 6 7 7 8 9 9
5 | 0 0 1 8
6 | 5
7 | 0 3

There are 3 types of center:

- Mean
  - $\bar{x} = \Sigma x/x$
- Median
  - After ordering, its the # in the middle
    (if two #s in the middle, take average)
- Mode
  - The number which occurs the most
    (if more than one → multimodal)
    (if none → no mode)

Ex 1, 2, 3, 3, 3, 6, 7, 9, 9, 10, 10

Mean

$\bar{x}$ = (1+2+3+3+3+6+7+9+9+10+10)/11

= 5.72727

Median

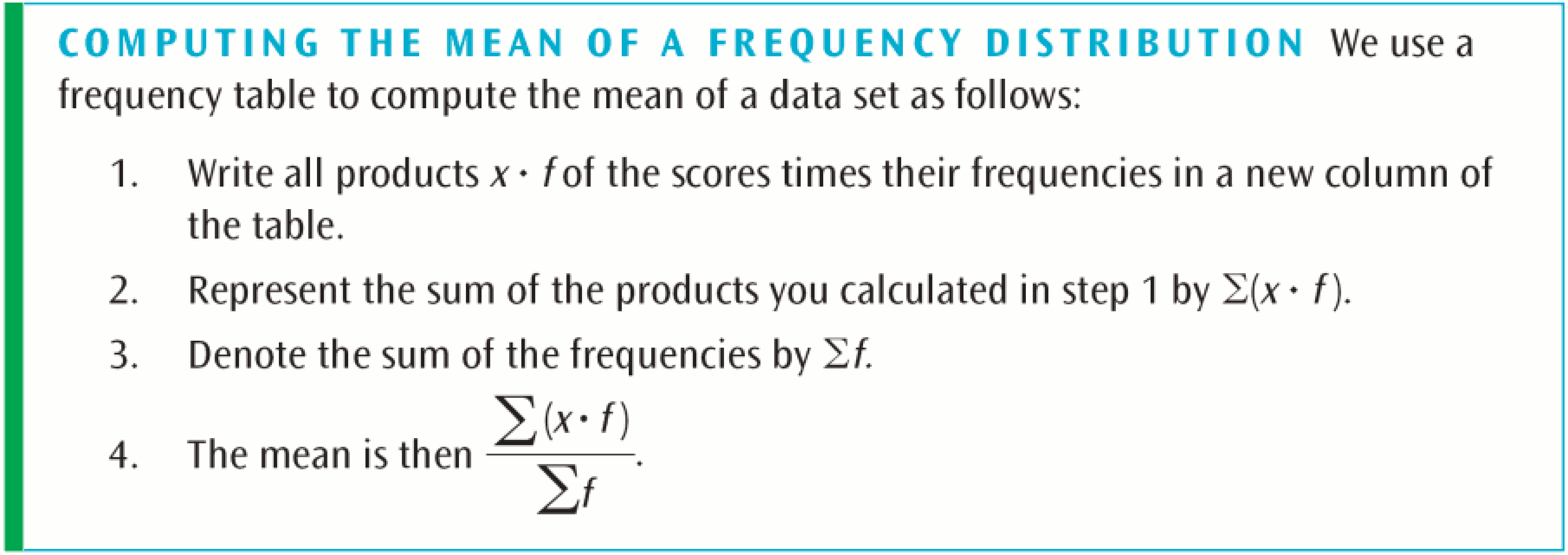
Middle = 6

Mode

Most = 3

Given a frequency table,

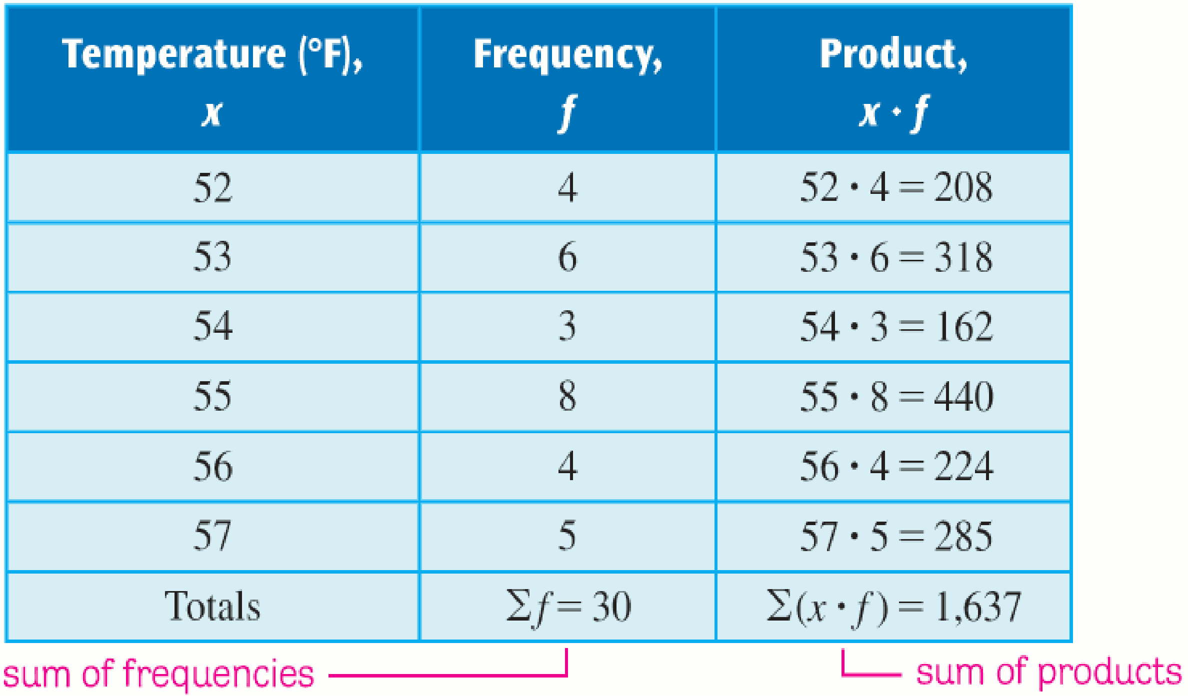
- find the total number of data points,
  which is the sum of the frequencies
  So find Σf
- find the sum of all values,
  if freq f occurs x times it contributes xf
  So find Σxf



COMPUTING THE MEAN OF A FREQUENCY DISTRIBUTION We use a frequency table to compute the mean of a data set as follows:

1. Write all products $x \cdot f$ of the scores times their frequencies in a new column of the table.
2. Represent the sum of the products you calculated in step 1 by $\Sigma(x \cdot f)$.
3. Denote the sum of the frequencies by Σf.
4. The mean is then $\dfrac{\sum(x \cdot f)}{\sum f}$.

Example: What is the mean temperature?



Temperature (°F), x	Frequency, f	Product, $x \cdot f$
52	4	$52 \cdot 4 = 208$
53	6	$53 \cdot 6 = 318$
54	3	$54 \cdot 3 = 162$
55	8	$55 \cdot 8 = 440$
56	4	$56 \cdot 4 = 224$
57	5	$57 \cdot 5 = 285$
Totals	$\Sigma f = 30$	$\Sigma(x \cdot f) = 1{,}637$

sum of frequencies

sum of products

The mean is:

$$\frac{\sum(x \cdot f)}{\sum f} = \frac{\text{sum of scores}}{\text{number of scores}} = \frac{1{,}637}{30} \approx 54.6°\text{F}.$$

Five Number Summary

1. Order the data

2. Find the smallest, largest and median.

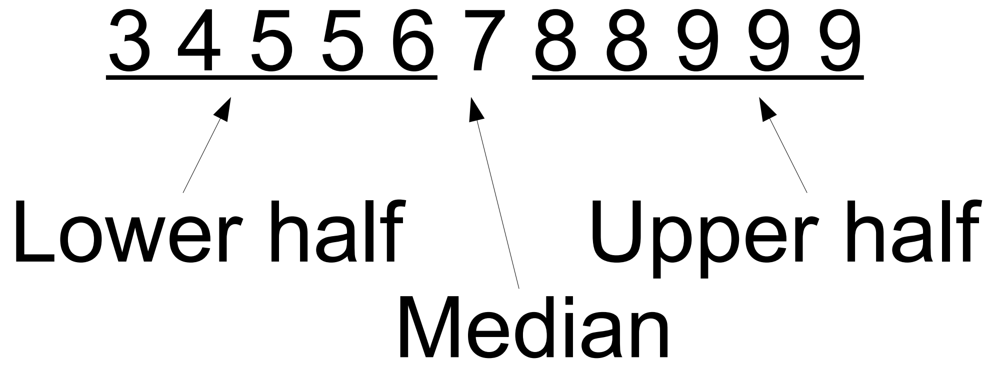
3. Find the median of the lower half, Q_1

4. Find the median of the upper half, Q_3

5. The Five Number Summary is:

smallest, Q_1, median, Q_3, largest

Example:



3 4 5 5 6 7 8 8 9 9 9

Lower half

Upper half

Median

Smallest = 3

Q_1 = 5

Median = 7

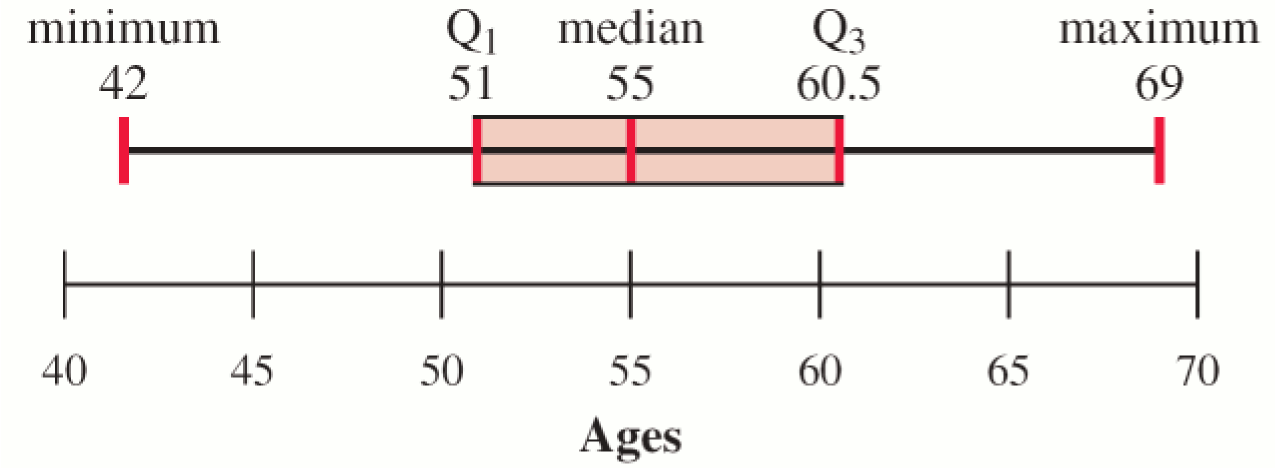
Q_3 = 9

Largest = 9

Summary: 3, 5, 7, 9, 9

The Box and Whiskers Plot is a visual representation of the Five Number Summary

Example: Summary = 42, 51, 55, 60.5 69



Definition:

$$\text{Std. Deviation} = \sqrt{\frac{\Sigma\,(x-\bar{x})^2}{n-1}}$$

Data x	$x-\bar{x}$	$(x-\bar{x})^2$
2	-3	9
4	-1	1
7	2	4
5	0	0
4	-1	1
8	3	9
Sum (Σ)	0	24

variance
= 24/5
= 1.2

std. dev.
= $(1.2)^{1/2}$
= 1.09544

Overall idea for Normal Distributions:

Raw data x ↔ z-score ↔ areas

use: $z = \frac{x - \mu}{\sigma}$ Table

μ (mu) = population mean

σ (sigma) = population standard deviation

Example:

Suppose the mean of a normal distribution is 20 and its std dev is 3.

Find the z-score of 25.

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{25 - 20}{3}$$

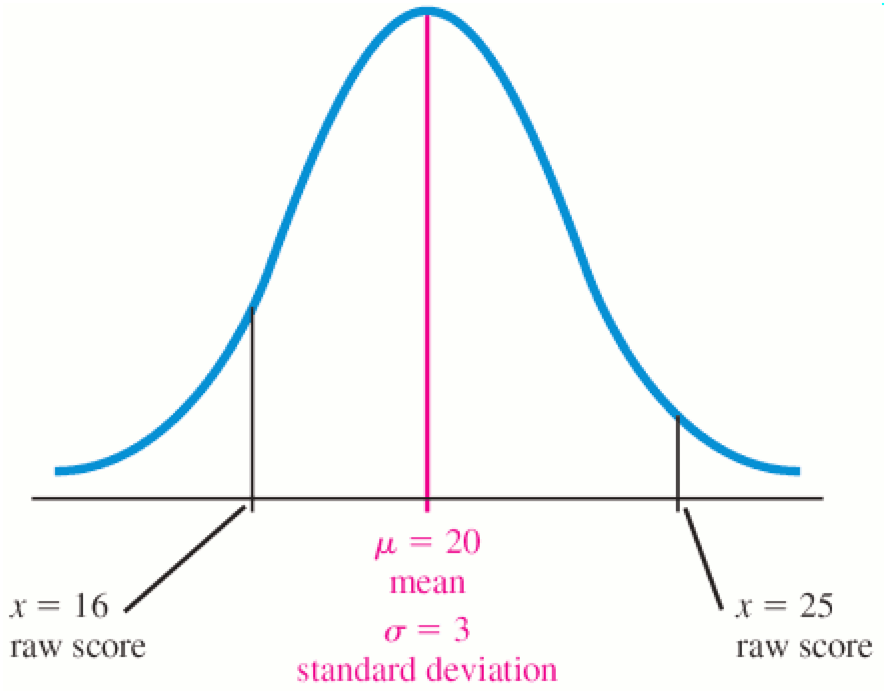
$$= \frac{5}{3} = 1.67.$$

Find the z-score of 16.

$$z = \frac{x - \mu}{\sigma}$$

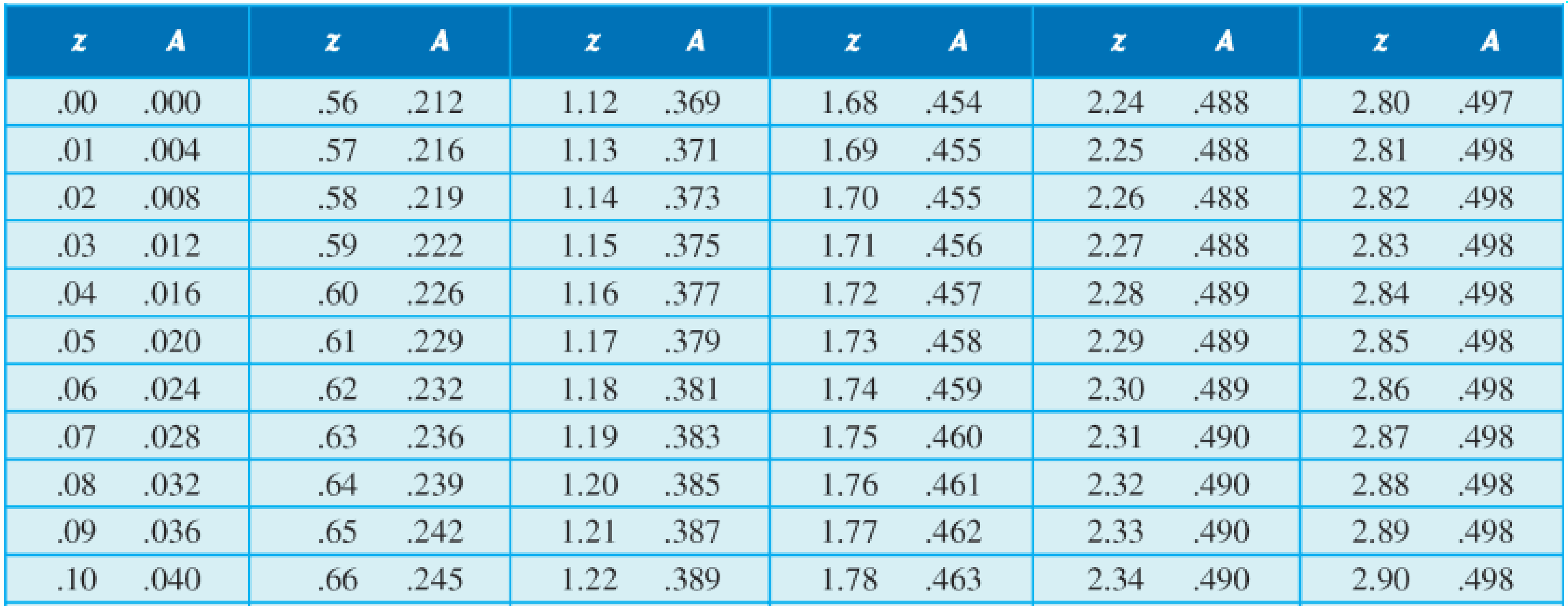
$$= \frac{16 - 20}{3}$$

$$= \frac{-4}{3} = -1.33.$$



The area between z-scores gives the percent of data values between them.

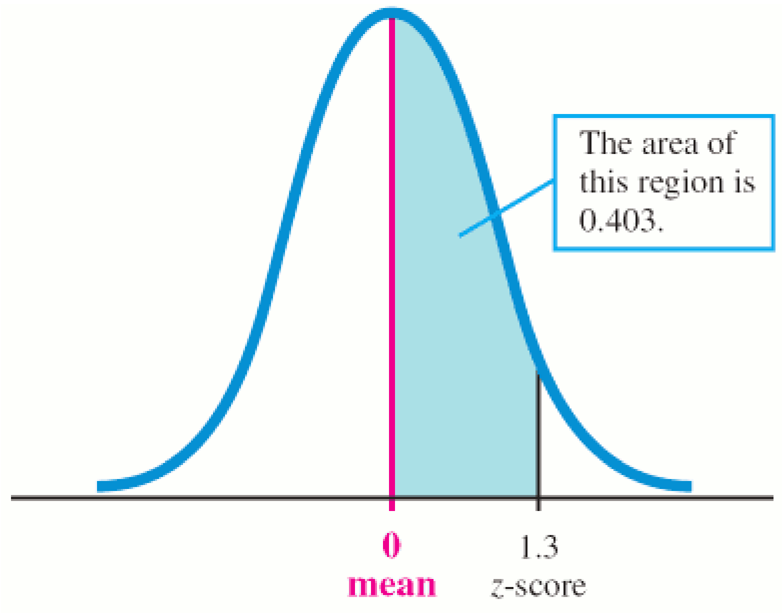
Below is a table that gives the area between the mean (μ) and a given z-score.



z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498

Find the percent of data between z=0 and z=1.3

For z = 1.3 A = 0.40



The area of this region is 0.403.

Area between = 0.403 or 40.3%